Heavy Quark Propagation in an AdS/CFT plasma

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Outline

Langevin dynamics for heavy quarks

Calibration of the noise

Broadening from Wilson Lines

AdS/CFT computation

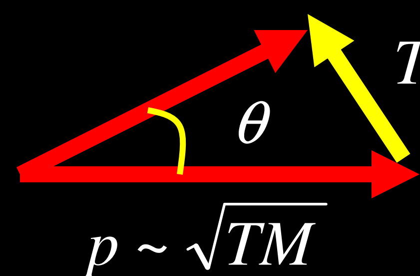
Broadening at finite velocity

Langevin Dynamics

Heavy Quark M>>T ⇒ moves slowly $\quad v_{th} \sim \sqrt{T/M}$

de Broglie w. l. $\quad \lambda \sim \dfrac{1}{\sqrt{MT}} << \dfrac{1}{T}$ $\quad$ *HQ is classical*

$$\frac{dp}{dt} = -\eta_D p + \xi(t)$$



$$\theta \sim \sqrt{\frac{T}{M}} << 1$$

Random (white) noise

$$\langle \xi(t), \xi(t') \rangle = \kappa\delta\big(\xi(t) - \xi(t')\big)$$

Einstein relations

Medium properties

$$\eta_D = \frac{\kappa}{2MT} \qquad D = \frac{2T^2}{\kappa}$$

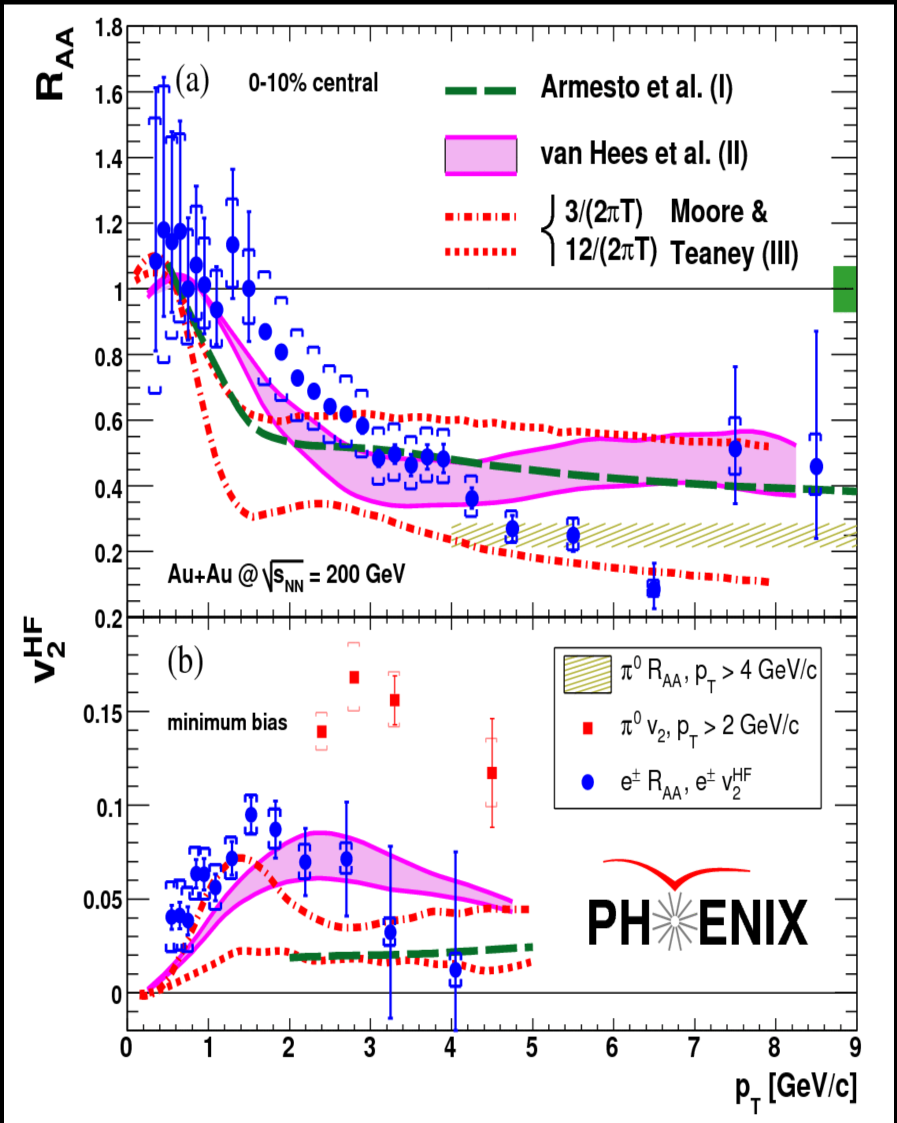
Heavy Quarks at RHIC

The Langevin model to is used to describe charm and bottom quarks

Fits to data allow to extract the diffusion coefficient

$$D = \frac{3-6}{2\pi T}$$

(Moore & Teaney, van Hees & Rapp)



How to Calibrate the Noise

In perturbation theory $\kappa = \left\langle p_T^2 \right\rangle \Big/ \tau_{col}$

But we cannot use diagrams!

M>>T $\Rightarrow$ long deflection time $\dfrac{1}{\eta_D} = \dfrac{M}{T}\dfrac{1}{D}$

$$\frac{dp}{dt} = -\eta_D p + \xi(t) \quad \underset{t \gg \tau_{med}}{\overset{t \ll 1/\eta_D}{\Longrightarrow}} \quad \frac{dp}{dt} = \xi(t) = F(t)$$

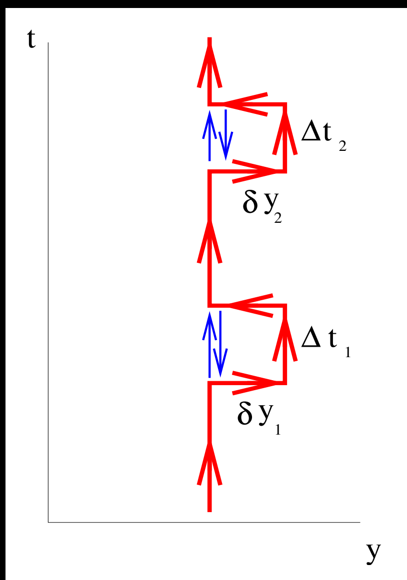
$q \quad E$

$$F(t) = \int d^3x \; \overline{Q}(t,x) T^a Q(t,x) \; E_a(t,x)$$

$\kappa = \int dt \left\langle F(t) F(0) \right\rangle$ Thermal average

Broadening from Wilson Lines

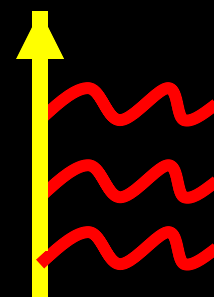
$$\kappa = \int_{t,x,x'} \left\langle \overline{Q}(t,x) T^a Q(t,x) E_a(t,x) \overline{Q}(0,x') T^a Q(0,x') E_a(0,x') \right\rangle$$



$$Q(t) = U(t,t_0)\,Q(t_0)$$

$E(t_1,y_1)\Delta t_1 \delta y_1$

$E(t_2,y_2)\Delta t_2 \delta y_2$



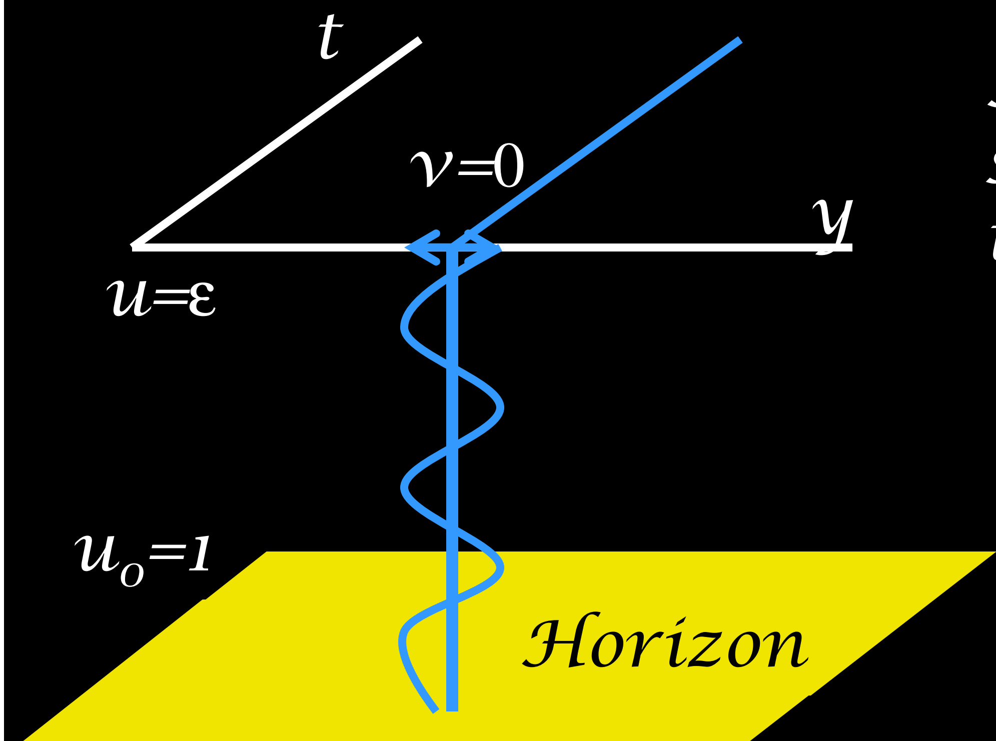
$$U(t,t_0) = e^{\,i\int_{t_0}^{t} A_\mu u^\mu d\tau}$$

Small fluctuation of the HQ path

$$W_C[\delta y] = T_C \exp\left\{ -i \int dt_C \left(A_0 + \delta\dot{y} A_y \right) \right\}$$

$$\kappa = \lim_{\omega \to 0} \int dt\, e^{+i\omega t} \frac{1}{\langle W_C[0,0] \rangle} \left\langle \frac{\delta^2 W_C[\delta y_1, \delta y_2]}{\delta y_2(t)\, \delta y_1(0)} \right\rangle$$

AdS/CFT computation



Static Quark:
straight string stretching
to the horizon

We solve the small
fluctuation problem

(mechanical problem)

$$\partial_u^2 y - \frac{2+6u^2}{4u\left(1-u^2\right)}\partial_u y + \frac{\omega}{4u\left(1-u^2\right)^2} y = 0$$

In terms of the classical solution

$$\kappa = \lim_{\omega\to 0} \frac{L^2}{\pi\alpha} \frac{2}{\pi\omega} \mathrm{Im} \frac{1}{u^{1/2}} y^*(\omega,u)\partial_u y(\omega,u) = g\sqrt{N_c}T^3\pi$$

Broadening and Diffusion in AdS/CFT

$$\kappa = g\sqrt{N_c}T^3\pi \qquad D = 2\big/\sqrt{\lambda}\pi T$$

In perturbation theory $\kappa \sim g^2 N$

Depends explicitly on N_c. Different from η/s

It is not universal!

Putting numbers

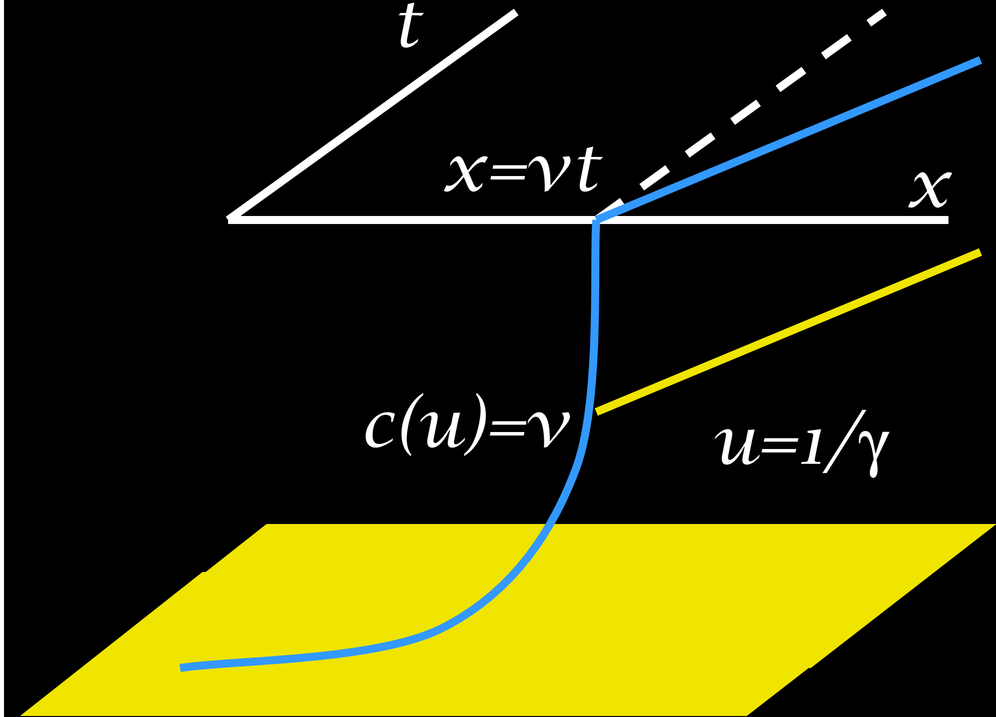
$$D \simeq \frac{1.0}{2\pi T}\left(\frac{1.5}{\alpha_{SYM} N}\right)^{1/2}$$

To compare with QCD => rescale the degrees of freedom

$$\frac{S_{SYM}}{S_{QCD}} = 2.4 \qquad \sqrt{\frac{S_{SYM}}{S_{QCD}}} = 1.6$$

(Liu, Rajagopal, Wiedemann)

Probe at finite velocity



Trailing string
=> work over tension

$$\frac{dp}{dt} = -\frac{\pi}{2}\sqrt{\lambda}T^2 \frac{v}{\sqrt{1-v^2}}$$

$$\eta_D = \frac{1}{2MT}\pi\sqrt{\lambda}T^3$$

same κ!

Drag valid for all p !

(Herzog, Karch, Kovtun, Kozcaz and Yaffe ; Gubser)

Broadening => repeat fluctuation problem

$$\kappa_T = g\sqrt{N_c\gamma}T^3\pi$$

Depends on the energy of the probe!

Conclusions

We have provided a "non-perturbative" definition of the momentum broadening as derivatives of a Wilson Line

This definition is suited to compute k in N=4 SYM by means of the AdS/CFT correspondence.

The calculated κ *scales as* $\sqrt{\lambda}$ *and takes much larger values than the perturbative extrapolation for QCD.*

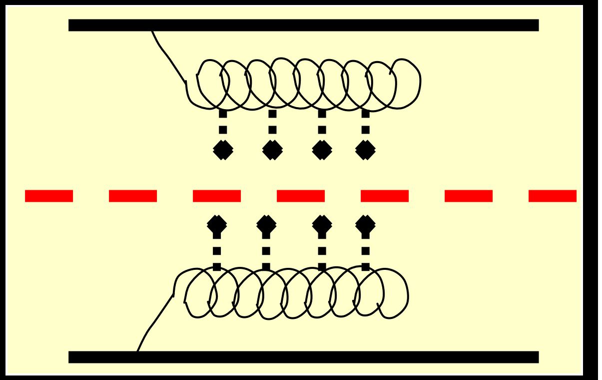
The results agree, via the Einstein relations, with the computations of the drag coefficient. This can be considered as an explicit check that AdS/CFT satisfy the fluctuation dissipation theorem.

The momentum broadening κ *at finite v diverges as* $\sqrt{\gamma}$.

Back up Slides

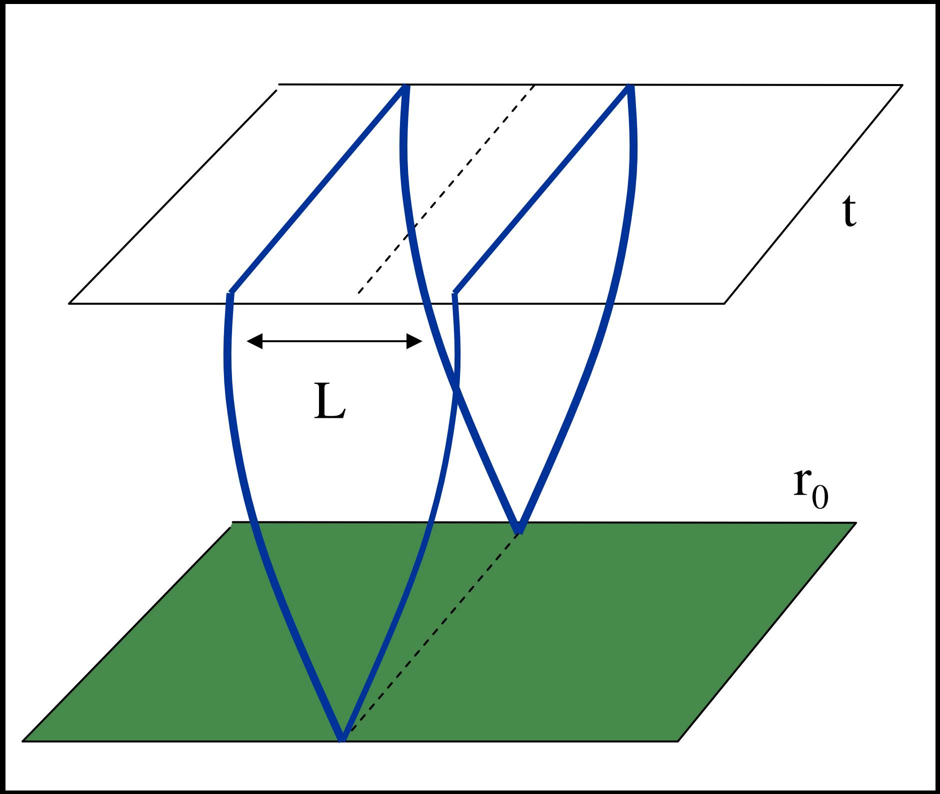
Computation of $\hat{q}$ (Radiative Energy Loss)

(Liu, Rajagopal, Wiedemann)



Dipole amplitude:

two parallel Wilson lines in the light cone:



Order of limits:

1) $v \to 1$

2) $M \to \infty$

String action becomes imaginary for $\gamma > \left(M/\sqrt{\lambda}T\right)^2$

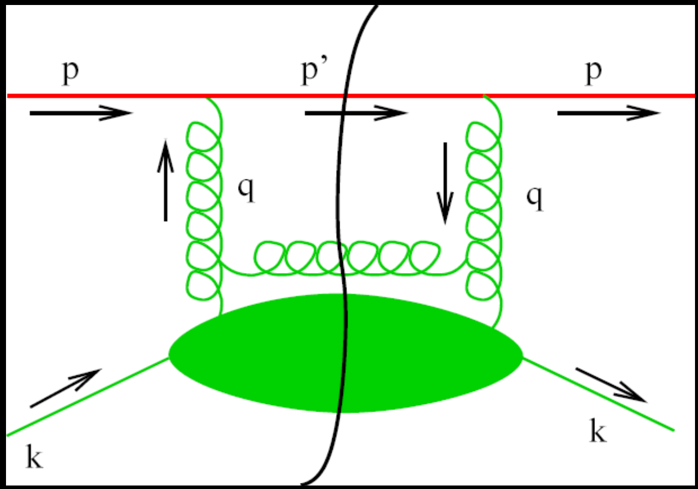
For small transverse distance:

$$\langle W \rangle = e^{-S} = e^{-\frac{1}{4}\hat{q}LL^-}$$

$$\hat{q}_{SYM} = 5.3\sqrt{g^2 N T^3} \quad \xRightarrow{\text{entropy scaling}} \quad \hat{q}_{QCD} \approx 6-12 \quad GeV^2/fm$$

Energy Dependence of $\hat{q}$

(JC & X. N. Wang)



From the unintegrated PDF

$$\hat{q}_R = \frac{4\pi^2\alpha_s C_R}{N_c^2-1}\rho\int_0^{\mu^2}\frac{d^2q_T}{(2\pi)^2}\int dx\delta(x-\frac{q_T^2}{2p^-\langle k^+\rangle})\phi(x,q_T^2)$$

Evolution leads to growth of the gluon density,

In the DLA $\phi\left(x,q_T^2\right)\sim e^{\sqrt{2\xi\left(q_T^2\right)\ln\frac{1}{x}}}$ $\xi\left(q_T^2\right)=\int\frac{dk^2}{k^2}\frac{2\alpha\left(q_T^2\right)N_c}{\pi}$

HTL provide the initial conditions for evolution.

Saturation effects $\Rightarrow$ $\hat{q}\sim Q_s^2\min\left(L,L_c\right)$

For an infinite conformal plasma ($L>L_c$) with $Q^2_{max}=6ET$.

$$\hat{q}_R = \frac{C_R}{C_A}T^3\left(\frac{Q^2_{max}}{T^2}\right)^{\sqrt{\frac{\bar{\lambda}}{2+\bar{\lambda}}}}\left(\frac{\pi^5}{72\sqrt{2\pi}}\frac{\rho}{(N_c^2-1)T^3}\bar{\lambda}^{5/4}(2+\bar{\lambda})^{1/4}\ln^{1/4}\frac{Q^2_{max}}{T^2}\right)^{\frac{2}{2+\bar{\lambda}}}$$
$$\left(\sqrt{2+\bar{\lambda}}-\sqrt{\bar{\lambda}}\right)^{\frac{4+\bar{\lambda}}{2+\bar{\lambda}}}\frac{1}{4}\left[\sqrt{\bar{\lambda}}+\frac{2}{\sqrt{2+\bar{\lambda}}+\sqrt{\bar{\lambda}}}\right].$$

At strong coupling

$$\hat{q}_R = \frac{3C_R}{2C_A}T^2E$$

Noise from Microscopic Theory

HQ momentum relaxation time: $\tau_{HQ} = \frac{1}{\eta_D} = \frac{M}{T} D >> \tau_{medium} \sim D$

Consider times such that $\tau_{HQ} >> \tau >> \tau_{medium}$

microscopic force (random)

$$\frac{dp}{dt} = -\eta_D p + \xi(t) \quad \Rightarrow \quad \frac{dp}{dt} = \mathcal{F}(t, \mathbf{x}) \leftarrow qE$$

charge density electric field

$$\mathcal{F} \equiv \int d^3x \, Q^\dagger(t, \mathbf{x}) T^a Q(t, \mathbf{x}) E_a(t, x)$$

$$\kappa = \int dt \, \langle \xi(t) \, \xi(0) \rangle = \int dt \, \langle \mathcal{F}(t, \mathbf{x}) \mathcal{F}(0, \mathbf{x}) \rangle_{HQ}$$

Heavy Quark Partition Function

McLerran, Svetitsky (82)

$$Z_{HQ} = \sum_s \langle s \left| e^{-\beta H} \right| s \rangle = \int d^3x \sum_{s'} \langle s' \left| Q(\mathbf{x}, -T) e^{-\beta H} Q^\dagger(\mathbf{x}, -T) \right| s' \rangle \ ,$$

$$= \int d^3x \sum_{s'} \langle s' \left| e^{-\beta H} Q(\mathbf{x}, -T - i\beta) Q^\dagger(\mathbf{x}, -T) \right| s' \rangle$$

YM + Heavy Quark states

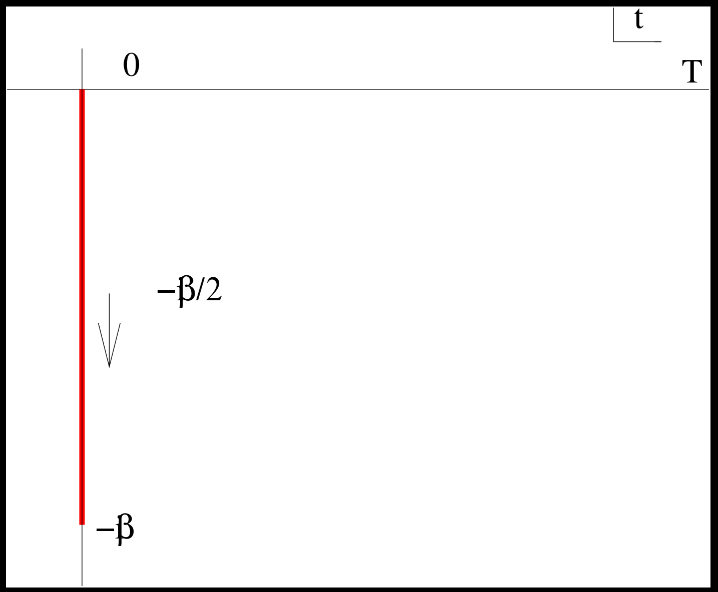
YM states

Integrating out the heavy quark

$$\mathcal{L} = Q^\dagger \left(i\partial_t - M - A_0 \right) Q$$

$$Z_{HQ} = V_{\mathrm{ps}} e^{-\beta M} \langle W_C[0] \rangle$$

Polyakov Loop



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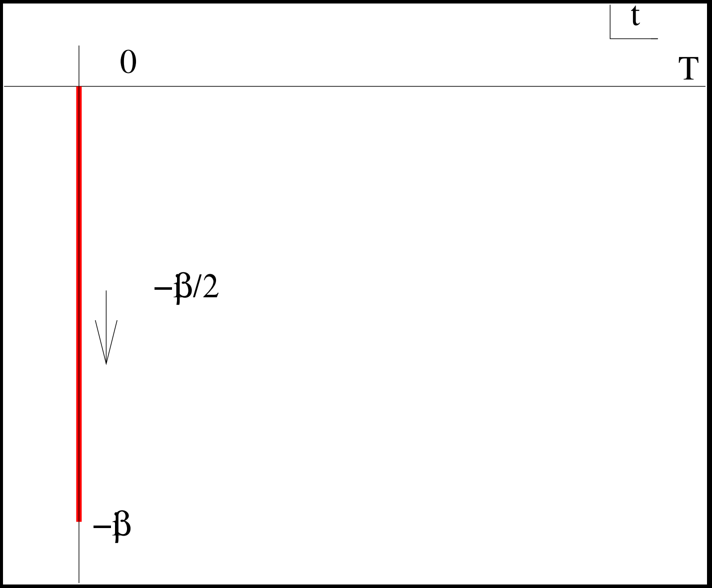


κ as a Retarded Correlator

κ is defined as an unordered correlator:

$$\kappa = \int dt \, \langle \mathcal{F}(t) \mathcal{F}(0) \rangle_{HQ}$$

From Z_{HQ} the only unordered correlator is

$$iG_{12}(t,t') = \langle \mathcal{F}_2(t') \mathcal{F}_1(t) \rangle_{HQ}$$

Defining:

$$iG_R(t) = \theta(t) \, \langle [\mathcal{F}(t), \mathcal{F}(0)] \rangle_{HQ}$$

$$\begin{aligned}
iG_{11}(\omega) &= i\mathrm{Re}\, G_R(\omega) - \coth\left(\frac{\omega}{2T}\right) \mathrm{Im} G_R(\omega) \\
iG_{12}(\omega) &= iG_{21}(\omega) = -\frac{2e^{\frac{-\beta\omega}{2}}}{1-e^{-\beta\omega}} \mathrm{Im} G_R(\omega) \\
iG_{22}(\omega) &= -i\mathrm{Re}\, G_R(\omega) - \coth\left(\frac{\omega}{2T}\right) \mathrm{Im} G_R(\omega)
\end{aligned}$$

In the ω→0 limit the contour dependence disappears :

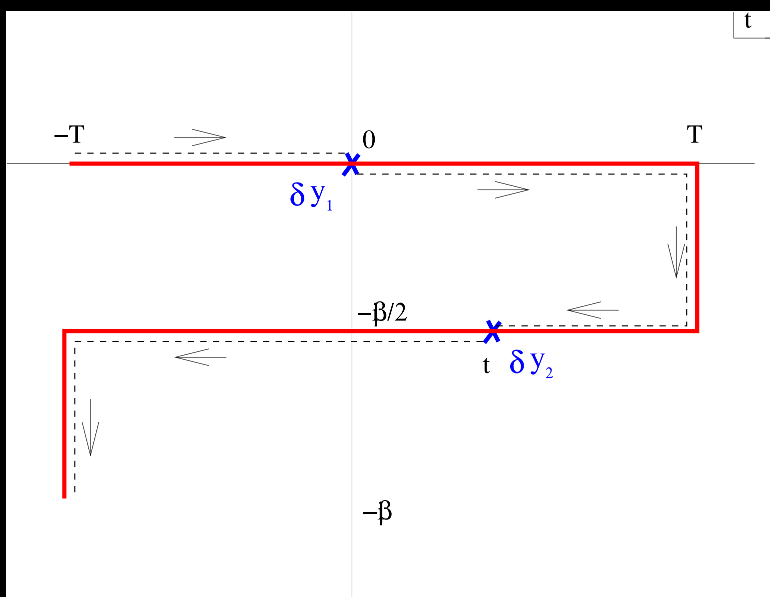
$$\kappa = \lim_{\omega \to 0} -\frac{2T}{\omega} \mathrm{Im} G_R(\omega)$$

Force Correlators from Wilson Lines

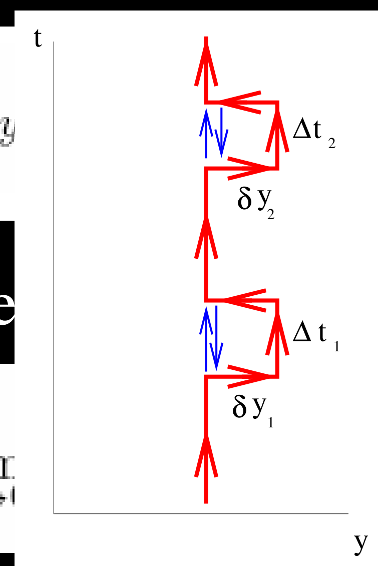
Integrating the Heavy Quark propagator: $\mathcal{F} \equiv \int d^3x\, Q^\dagger(t,\mathrm{x}) T^a Q(t,\mathrm{x})\, E_a(t,x)$

$$\langle T_C[\mathcal{F}(t_C)\mathcal{F}(0)]\rangle_{HQ} = \frac{1}{\langle W_C[0]\rangle} \langle \mathrm{tr}[U(-T-i\beta, t_C)\, E(t_C)\, U(t_C,0)\, E(0)\, U(0,-T)]\rangle$$

Which is obtained from small fluctuations of the Wilson line



$W_C[\delta y$ … $dt_C\, (A_0 + \delta y A_y)\Big\}$ V_1

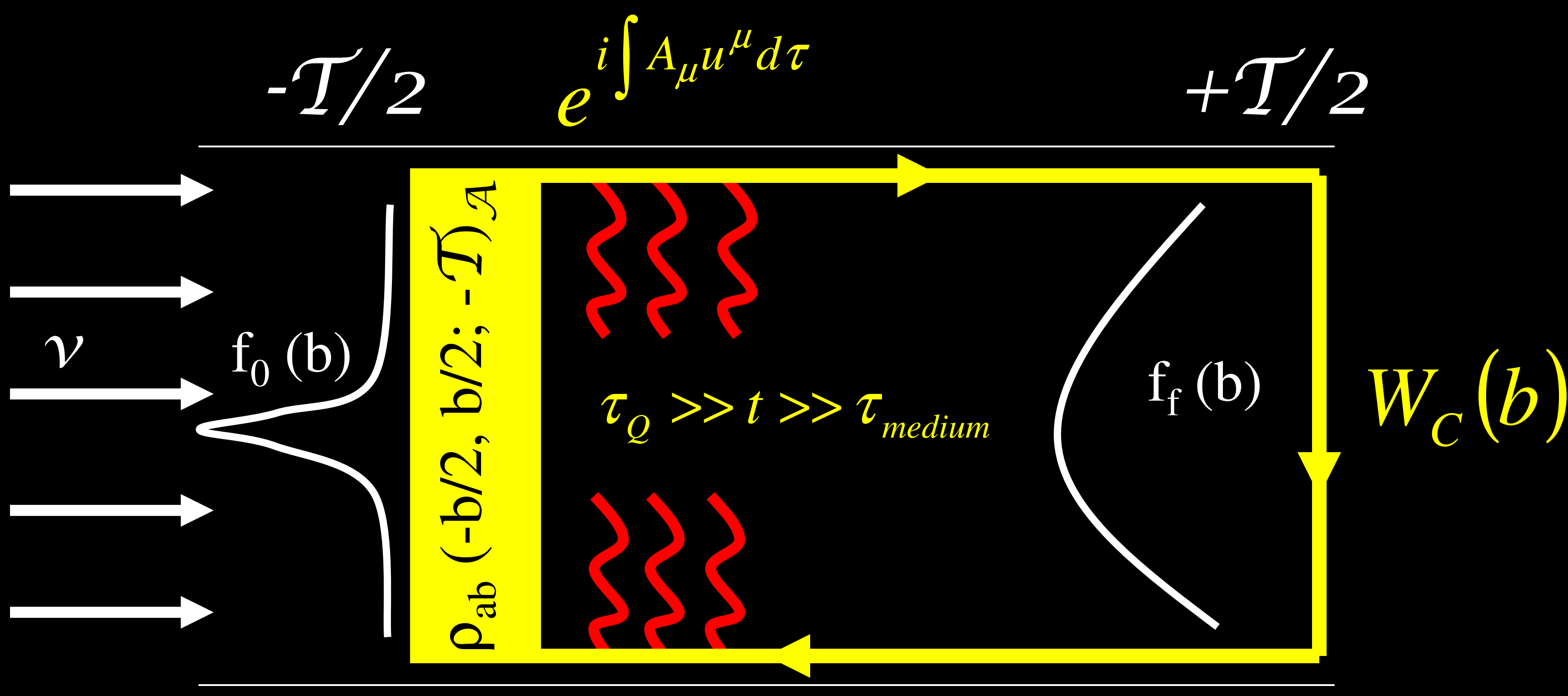


Since … o time order:

$E(t_2, y_2)\Delta t_2 \delta y_2$

$\kappa = \lim_{\omega\to}$ … $,0]\rangle \left\langle \frac{\delta^2 W_C[\delta y_1, \delta y_2]}{\delta y_2(t)\, \delta y_1(0)} \right\rangle$

Momentum Distribution



Final distribution

$$f_f(b) = \langle Tr[\rho(b) W_c(b)] \rangle_A$$